A Cloak for the Dreamer

We have just read the book entitled, *A Cloak for the Dreamer*. Your task is to create a fabric swatch design that follows these guidelines:

A) The design must use at least 4 different shapes.
B) The shapes must tessellate.
C) Your design must be colorful.
D) Your swatch must fill an 8 1/2” x 11” sheet of paper.
E) Your design must be a repeatable pattern.

Now, for the tough part. The tailor has asked that you create your design on a bolt of fabric that can then be cut and used to make new cloaks for the archduke. Each shape would be cut out and sewn together to create this bolt of fabric in the design you have created on your swatch. A bolt of fabric is 8 yards long, by 45 inches wide.

Mathematically determine the number of each shape you would need to cut and sew.

Write the tailor a letter telling how you determined how many of each shape you would need.

Try to provide him with directions that he can use to determine the number of shapes he would need to cut, in making your design using any size fabric.

Be sure to use a math representation, use math language, and make a mathematical connection. Also, be sure to show all of your work!
Grade Level 6–8

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Context

I had just read students the children’s book entitled *A Cloak for the Dreamer*, about a tailor who asks his three sons to help him make cloaks for an archduke. Two of the sons are able to create cloaks with shapes that tessellate, while the third son creates a cloak made of circles that do not tessellate.

The third son decides he is not made for this line of work and sets out to visit the world instead. His father and two brothers cut the circles in his cloak into hexagons, and give it to him as a “going away” gift to keep him safe.

I provided direct instruction on how to use algebra to create and extend patterns. We spent about two weeks doing algebraic activities (see Teaching Tips), before this task was presented.
**What This Task Accomplishes**

This task assesses how well students can create patterns that tessellate a plane, and then create a formula for finding how many of each shape would tessellate a plane of any area.

**Time Required for Task**

This task took about a week of math class time to complete.

**Interdisciplinary Links**

This task could link well to studies of fabric and patterns in family and consumer science class. In fact, students could design their own fabric and use that fabric to create a piece of clothing, a pillow, or whatever they would like.

**Teaching Tips**

Most of my students used pattern blocks to create a pattern. They then traced their patterns onto 8.5” x 11” sheets of paper. Students then created a function table to solve for the number of shapes needed for 1, 2, 3 and “n” sections. Student then determined the area of one section of their pattern and divided that into the area of the bolt of fabric. They then substituted the value into their tables and solved for “n” number of sections.

To motivate students to solve the task, I read to them the beautiful picture book, *A Cloak for the Dreamer*. After doing many activities with patterns I designed, students were excited to create designs of their own. I decided students needed instruction on how to use an algebraic formula to determine the number of pieces needed in similar tasks.

<table>
<thead>
<tr>
<th>Section #n Pieces</th>
<th>Squares</th>
<th>Triangles with Patterns</th>
<th>Plain Triangles</th>
<th>Total Number of Pieces</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2</td>
<td>2</td>
<td>1</td>
<td>5</td>
</tr>
<tr>
<td>2</td>
<td>3</td>
<td>4</td>
<td>2</td>
<td>9</td>
</tr>
<tr>
<td>3</td>
<td>4</td>
<td>6</td>
<td>3</td>
<td>13</td>
</tr>
<tr>
<td>n</td>
<td>n+1</td>
<td>2n</td>
<td>n</td>
<td>4n+1</td>
</tr>
</tbody>
</table>
Students did lots of activities like this before being given this fabric design task.

This task can easily be modified in many different ways for students with special needs. Some ways of modifying this task are to:

- modify the size of the bolt of fabric or omit this part from the task altogether.
- provide students with an already made section using four different shapes.
- limit the number of shapes.
- provide a shape that has an easily determined area that can easily fit into a larger size area without remainders to deal with.
- require that students just create a shape the tessellates and can fill an 8.5” x 11” piece of paper.
- photograph the shape the student creates using pattern blocks rather than having the student trace it.
- allow the student to use pattern block stamps or stickers to record their designs on paper.

The modifications chosen would depend on individual student ability and need. For most students in my classroom, modifying the task so that the bolt of fabric was smaller and didn’t require the converting of units of measure was a sufficient modification that enabled them to access the task.

To make assessment easier, you could ask students with similar solutions to hand in their work together so you can look at the correctness of one type of pattern at a time. After spending a few days working on the task, my students brainstormed a list of ideas they could use to extend their solutions and to come up with mathematical connections. Here is the list of some of the ideas they came up with:

- Redefine what a section is, solve the problem again, and see how it affects the answer.
- Convert to metric units of measure.
- Determine the percent or fraction of each shape you used as it relates to the whole.
- Describe how this task is mathematically similar to another.
- Assign each shape a price and determine the overall cost of the bolt of fabric based on that price.
- Identify patterns or rules that can be used in solving the problem.
- Identify the underlying mathematics of the task.

**Suggested Materials**

Provide pattern blocks, paper, calculators, rulers, and graph paper.
Possible Solutions

Solutions will vary depending on the individual design the student creates. When assessing correctness, look to see that the student used four shapes that tessellate, that s/he correctly found the number of shapes that would be needed for the bolt of fabric, and that all measurements, data tables, and computations are accurate.

Benchmark Descriptors

Novice

The novice will not be able to create a design that repeats and tessellates using four shapes; or this may be as far as the student can get in solving the task as s/he doesn’t have a strategy on where to go from there. The novice will use little or no math language, and representations will lack labels and/or accuracy. Little reasoning will be evident.

Apprentice

The apprentice will create a design that repeats and tessellates, but then is not able to reach a correct solution because of a computation error, an error in his/her data table, a measuring error, or a reasoning error. Some math language will be used, and a math representation will be attempted and accurate to the student’s solution.

Practitioner

The practitioner will create a design that repeats and tessellates, and then is able to use that design to determine the number of each shape that would be needed when creating a bolt of fabric. Accurate and appropriate math language will be used, and math representations will be accurate and labeled. Reasoning throughout will be correct, and the solution easy to follow.

Expert

The expert will create a design that repeats and tessellates, and then is able to use that design to determine the number of each shape that would be needed when creating a bolt of fabric. Sophisticated use of math language will be used, and math representations will be accurate and labeled. Reasoning throughout will be correct and will demonstrate sophistication. The approach and reasoning will be explicated, and the student will make mathematically relevant comments and observations about the solution.
Author

Carol McNair teaches grade six at the Camels Hump Middle School in Richmond, Vermont. She has worked as the mathematics assessment consultant for the Vermont Department of Education’s Portfolio assessment program, and acts as a consultant to school districts and to publishing companies. She is also the editor of Math Exemplars.
Dear Tailor,

I was pleased that you chose me to make a bolt of fabric for the archduke so you will be able to make a cloak for the archduke. First, I made a pattern of 7 shapes: hexagons, trapezoids, squares, and rhombuses. Then I made a section of the shapes which I outlined in black. Afterwards I solved for X. That held me back a little bit, Mr. Tailor. Fourthly, I found out the area of the bolt of fabric. I found out the numbers of shapes in the one section. Then I found out the number of shapes in the whole bolt of fabric. For my math connections, Mr. Tailor, are a little confusion. I figured out theisoft the bolt, and I also figured out the total # of pieces adding all that would be in the bolt of fabric.

Once again thank you for choosing me for the job.

The student attempts to explain the approach used.
Novice (cont.)

The student attempts a connection, but it is incorrect.

Few parts of this solution are correct. The reader is able to follow some of what the student did.

Some math language is used.

This student is able to create a math representation which represent her/his incorrect solution. Many novices may be unable to do this.

Parts of this table are correct but much of it is not. The strategy would work if executed properly.
The student is unable to create a pattern that correctly tessellates.
Apprentice

The student finds the area of the paper, and s/he should have found the area of one section. This is a flaw that leads to an incorrect solution.

The student creates an accurate and appropriate table to find for “N” number of sections.

Correct math language is used throughout.

The student confuses the size of his/her paper with the size of the bolt of fabric.

Dear Tailor,

I have found a new design for a cloak. I also found out that it would take 698 shapes to fill in a yard by an inch sheet of 40th. The way I’ve determined this answer is that first derived a chart shown on the top left of this sheet.

I first listed all of the shapes for the first section. Then the second and third. By using the chart pattern on the chart I was able to determine the formula for N.

I then measured the measurement of the cloth and put it into simplest terms. Then by multiplying the measurement of the cloth in inches. I was able to equal 12,960 square inches.

I did the same to the measurement of the "Swatch" which equaled 93.5 square inches. By dividing the 12,960 by 93.5 I received the answer of 139.

The pattern found in the chart and the new formula. I was able to devise an answer of 698.

So I now that 698 is the number of shapes needed for the 8 yard by 45 inch sheet of cloth and by using my formula, you will be able to use my pattern in any shape of cloth. For a new cloak for a dreamer.
Apprentice (cont.)

The student is able to create a design that tessellates.
Dear tailor,

I found that using my pattern; for one bolt of fabric you will need 1,154 hexagons, 2,304 trapezoids, 1,152 rhombuses, and 2,304 triangles. I determined this by measuring one section of my pattern width wise and length wise, and then knowing this information I measured how many sections fit width wise on the bolt and how many length wise. 48 fit width wise and 12 fit length wise. So I multiplied these 2 numbers and came up with 576 sections will fit in 1 bolt. Next I found out that there were 4 hexagons, 4 trapezoids, 2 rhombuses, and 4 triangles in 1 pattern. Next I made a chart similar to the one below to find out how many of each shape I need total for 1 bolt.

<table>
<thead>
<tr>
<th>Section</th>
<th>Hexagons</th>
<th>Trapezoids</th>
<th>Rhombuses</th>
<th>Triangles</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>4</td>
<td>4</td>
<td>2</td>
<td>4</td>
<td>14</td>
</tr>
<tr>
<td>2</td>
<td>8</td>
<td>8</td>
<td>4</td>
<td>8</td>
<td>26</td>
</tr>
<tr>
<td>3</td>
<td>12</td>
<td>12</td>
<td>6</td>
<td>12</td>
<td>38</td>
</tr>
<tr>
<td>N</td>
<td>2N + 2</td>
<td>4N</td>
<td>2N</td>
<td>4N</td>
<td>12N + 2</td>
</tr>
<tr>
<td></td>
<td>576</td>
<td>1,154</td>
<td>2,304</td>
<td>1,152</td>
<td>6,914</td>
</tr>
</tbody>
</table>

where:

\[N = \text{Section}\]
\[2N = \text{Section multiplied by 2}\]
\[4N = \text{Section multiplied by 4}\]
\[12N = \text{Section multiplied by 12}\]
\[N + 2 = \text{Section plus 2}\]

So if you would like to use my pattern on different sized fabric just use the above formula. The total number of pieces in the bolt is 6,914. This is like the tile problem because I had to design a colorful pattern for both problems. If it cost 25 cents for each hexagon, 20 for each trapezoid, 18 for each rhombus, and 10 for each triangle it would cost a total of 1 bolt of fabric which would be $1,187.06 dollars. The total triangle cost would be $230.40, the hexagon cost would be $288.50, the total rhombus cost would be $207.36, and the total trapezoid cost would be $460.80.
The student creates a pattern that tessellates.
I first started out by drawing a "section" and making a chart. I figured out sections 1, 2, 3, and N (number of sections). After that I divided 388 (the number of inches in the bolt of cloth) by 3 (the length of 1 section). Here is an algebraic equation: \( V(x) = \frac{z}{W} \)  

\[ \text{Where:} \quad \begin{align*}  V &= \text{V (number of yards in a bolt)} \\  x &= 36 \text{ (number of inches in a yard)} \\  z &= \text{number of inches in a bolt (388 inches)} \\  y &= \text{the length of 1 section (6 inches)} \\  w &= \text{total # of sections in a bolt.} \end{align*} \]

Then I found the number of blocks in the total number of sections using the \( N \) formulas. I repeated this step for the second type of pattern. Once I found the lengths and widths I multiplied them together and found the total of blocks.

All work is shown and labeled.

The student’s approach and reasoning are explained.

The student has a logical approach presented in a logical manner.

Accurate and appropriate math language is used throughout.
The student makes a connection by extending his/her solution. The student could have used this to verify the solution, but s/he does not.

The student makes another mathematically relevant connection.

For my connection I decided to look at my pattern, and another type of section, and then determine which pattern uses more blocks, less blocks, or the same amount. I went through the same process as the first patterns. I found the length and width and then multiplied them together. I figured out that there is no difference between the two sections.

On my original pattern I had a square border. Since it would normally be a separate bolt of fabric in a store I solved the area separately. Above is all my work. I used the same process as the others, but figured it out only using length since in a store the width of the pattern would be the width of the bolt of fabric.
The student creates a design using four shapes that tessellate the plane along with a border.